Limites de fonctions Polynômes en ∞

- $P(x)=a_n x^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+\cdots+a_1x+a_0$

$$\lim_{x\to\infty}P(x)=\lim_{x\to\infty}a_nx^n$$

Limites de fonctions Rationnelle en ∞

$$f(x)=\frac{P(x)}{Q(x)}=\frac{a_n x^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+\cdots+a_1x+a_0}{b_m x^m+b_{m-1}x^{m-1}+b_{m-2}x^{m-2}+\cdots+b_1x+b_0}$$

$$\lim_{x\to\infty}f(x)=\lim_{x\to\infty}\frac{P(x)}{Q(x)}=\lim_{x\to\infty}\frac{a_nx^n}{b_mx^m}$$

Limites de fonctions du type $\frac{f(x)}{g(x)}$ *en* a *telle que :* $f(a)=g(a)=0$

- $\lim\limits_{x\to2}\dfrac{-x^3+2x^2+x-2}{-x^2+5x-6}$
- $\lim\limits_{x\to1}\dfrac{x^3+2x-3}{x^2+2x-3}$
- $\lim\limits_{x\to1}\dfrac{x^3+x-2}{x^3-1}$
- $\lim\limits_{x\to2}\dfrac{x^3-6x^2+11x-6}{x^5-32}$
- $\lim\limits_{x\to1}\dfrac{\sqrt{2x-1}-\sqrt{x^2-x+1}}{\sqrt{x}-1}$
- $\lim\limits_{x\to9}\dfrac{\sqrt{x+7}-\sqrt{x}-1}{\sqrt{x+16}-5}$
- $\lim\limits_{x\to3}\dfrac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^3-27}$
- $\lim\limits_{x\to3}\dfrac{\sqrt{x+1}-x^2+x+4}{\sqrt{x+1}-\sqrt{3x+1}}$

Limites de fonctions et ordre

Rappel

$|\sin x|\le1$; $|\cos x|\le1$; $x\le E(x)<x+1$

- $\lim\limits_{x\to x_0^+}E(x)=x_0$; $\lim\limits_{x\to x_0^-}E(x)=x_0-1$

- *Soit a un réel ou infini*

$$\begin{cases} f(x) \geq g(x) \\ \lim\limits_{x \to a} g(x) = +\infty \end{cases} \Rightarrow \lim\limits_{x \to a} f(x) = +\infty$$

$$\begin{cases} g(x) \leq f(x) \leq h(x) \\ \lim\limits_{x \to a} g(x) = \lim\limits_{x \to a} h(x) = l \end{cases} \Rightarrow \lim\limits_{x \to a} f(x) = l \text{ (l un réel ou infini)}$$

$$\begin{cases} |f(x) - l| \leq g(x) \\ \lim\limits_{x \to a} g(x) = 0 \end{cases} \Rightarrow \lim\limits_{x \to a} f(x) = l \text{ (où l est un réel)}$$

Limites de fonctions trigonométriques

- $\lim\limits_{x \to 0} \dfrac{\sin(x)}{x} = 1$
- $\lim\limits_{x \to 0} \dfrac{1 - \cos(x)}{x^2} = \dfrac{1}{2}$
- $\lim\limits_{x \to 0} \dfrac{\tan(x)}{x} = 1$

Exercice

Calculer $\lim\limits_{x \to 0} \dfrac{\sin(5x)}{7x}$ *et* $\lim\limits_{x \to 0} \dfrac{\sin(5x)}{\sin(7x)}$.

Exercice

Calculer $\lim\limits_{x \to 0} \dfrac{\tan(7x)}{5x}$ *et* $\lim\limits_{x \to 0} \dfrac{\sin(5x)}{\tan(7x)}$.

Exercice

Calculer :

1) $\lim\limits_{x \to 0} \dfrac{\sin^2(x)}{\tan^2(x)}$

2) $\lim\limits_{x \to 0} \dfrac{\sin^2(x)}{1 - \tan^2(x)}$.

Exercice

Calculer : $\lim\limits_{x \to 0} \dfrac{1 - \cos(x)}{x}$ *puis* $\lim\limits_{x \to 0} \dfrac{\sin(x) - \cos(x) + 1}{x}$.

Exercice

Calculer :

1) $\lim\limits_{x \to 0^+} \dfrac{\sin(2x)}{\sqrt{1 - \cos(x)}}$

2) $\lim\limits_{x\to 0}\dfrac{\tan(x)-\sin(x)}{x^3}$.

Exercice

Calculer :

1) $\lim\limits_{x\to\frac{\pi}{2}}\left(\sin(x)-1\right)\tan^2(x)$

2) $\lim\limits_{x\to\pi}\left(1+\cos(x)\right)\tan\left(\dfrac{x}{2}\right)$.

Exercice

Calculer :

1) $\lim\limits_{x\to\frac{\pi}{2}}\left(\sin(x)-1\right)\tan^2(x)$

2) $\lim\limits_{x\to\pi}\left(1+\cos(x)\right)\tan\left(\dfrac{x}{2}\right)$.

Exercice

Calculer :

1) $\lim\limits_{x\to\alpha}\dfrac{\cos(x)-\cos(\alpha)}{\sin(x)-\sin(\alpha)}$

2) $\lim\limits_{x\to\alpha}\dfrac{\tan(x)-\tan(\alpha)}{\cos(x)-\cos(\alpha)}$.

2) $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\sin(3x)}{1-2\cos(x)}$.