Série d'exercices sur « limites et continuité »

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Exercice 1 :

Calculer les limites suivantes :

- $\lim\limits_{x\to-\infty} \dfrac{\sqrt{a^2x^2+1}-ax+2}{1-3a+ax}$ *où* $a\in\mathbb{R}_+^*$
- $\lim\limits_{x\to+\infty} \dfrac{\sqrt{x+3}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x+1}}$
- $\lim\limits_{x\to0^+} \dfrac{\tan x\sqrt{\tan x}-\sin x\sqrt{\sin x}}{x^3\sqrt{x}}$
- $\lim\limits_{x\to0} \dfrac{\sqrt{1-\cos^2 x}}{1-\cos x}$
- $\lim\limits_{x\to\frac{\pi}{4}} \tan(2x).\tan\left(x-\dfrac{\pi}{4}\right)$
- $\lim\limits_{|x|\to+\infty} \dfrac{x}{x^2+1}.\cos x$
- $\lim\limits_{x\to-1} \dfrac{\sin\left(\cos\dfrac{\pi}{2}x\right)}{x+1}$.
- $\lim\limits_{x\to0} \dfrac{\sin\left(\pi\sqrt{\cos x}\right)}{x}$.
- $\lim\limits_{x\to\frac{\pi}{3}} \dfrac{\sin x-\sqrt{3}\cos x}{x-\dfrac{\pi}{3}}$
- $\lim\limits_{x\to a} \dfrac{ax^n-a^nx}{x-a}$ *où* $a\in\mathbb{R}$
- $\lim\limits_{x\to2} \dfrac{\sum\limits_{k=1}^{n} x^k-2^{n+1}+2}{(3-x)^{n+1}-1}$
- $\lim\limits_{x\to1} \dfrac{x^{4n+1}\sqrt{x+3}+2\sqrt{x}-4}{1-x^{3n+2}}$

Correction

► $\lim\limits_{x\to-\infty}\dfrac{\sqrt{a^2x^2+1}-ax+2}{1-3a+ax}$ *où* $a\in\mathbb{R}_+^*$

Pour $x<0$ *et* $x\neq\dfrac{3a-1}{a}$ *; on a :* $\dfrac{\sqrt{a^2x^2+1}-ax+2}{1-3a+ax}=\dfrac{\sqrt{a^2x^2+1}-ax}{1-3a+ax}+\dfrac{2}{1-3a+ax}$

$$=\frac{|ax|\sqrt{1+\frac{1}{a^2x^2}}-ax}{-ax\left(-\frac{1}{ax}+\frac{3a}{ax}-1\right)}+\frac{2}{1-3a+ax}$$

$$=\frac{-ax\sqrt{1+\frac{1}{a^2x^2}}-ax}{-ax\left(\frac{3a-1}{ax}-1\right)}+\frac{2}{1-3a+ax}$$

$$=\frac{\sqrt{1+\frac{1}{a^2x^2}}+1}{\left(\frac{3a-1}{ax}-1\right)}+\frac{2}{1-3a+ax}$$

Donc $\lim\limits_{x\to-\infty}\dfrac{\sqrt{a^2x^2+1}-ax+2}{1-3a+ax}=\lim\limits_{x\to-\infty}\left(\dfrac{\sqrt{1+\frac{1}{a^2x^2}}+1}{\left(\frac{3a-1}{ax}-1\right)}+\dfrac{2}{1-3a+ax}\right)$

Et $\lim\limits_{x\to-\infty}\left(\dfrac{2}{1-3a+ax}\right)=0$; $\lim\limits_{x\to-\infty}\sqrt{1+\dfrac{1}{a^2x^2}}=1$; $\lim\limits_{x\to-\infty}\left(\dfrac{3a-1}{ax}\right)=0$

Alors $\lim\limits_{x\to-\infty}\dfrac{\sqrt{a^2x^2+1}-ax+2}{1-3a+ax}=-2$

► $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x+3}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x+1}}$

Soit $x>0$ *; on a :* $\dfrac{\sqrt{x+3}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x+1}}=\dfrac{1}{\dfrac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x+3}}+\dfrac{\sqrt{x+1}}{\sqrt{x+3}}}$

$$=\frac{1}{\sqrt{\frac{x+\sqrt{x+\sqrt{x}}}{x+3}}+\sqrt{\frac{x+1}{x+3}}}$$

$$=\frac{1}{\sqrt{\frac{x}{x+3}+\frac{\sqrt{x+\sqrt{x}}}{x+3}}+\sqrt{\frac{x+1}{x+3}}}$$

$$=\frac{1}{\sqrt{\frac{x}{x+3}+\frac{\sqrt{\frac{1}{x}+\frac{1}{x\sqrt{x}}}}{\left(1+\frac{3}{x}\right)}}+\sqrt{\frac{x+1}{x+3}}}$$

Donc $\lim\limits_{x\to+\infty}\frac{\sqrt{x+3}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x+1}}=\lim\limits_{x\to+\infty}\frac{1}{\sqrt{\frac{x}{x+3}+\frac{\sqrt{\frac{1}{x}+\frac{1}{x\sqrt{x}}}}{\left(1+\frac{3}{x}\right)}}+\sqrt{\frac{x+1}{x+3}}}$

Or $\lim\limits_{x\to+\infty}\frac{x}{x+3}=1$; $\lim\limits_{x\to+\infty}\frac{\sqrt{\frac{1}{x}+\frac{1}{x\sqrt{x}}}}{\left(1+\frac{3}{x}\right)}=0$ *et* $\lim\limits_{x\to+\infty}\sqrt{\frac{x+1}{x+3}}=1$

Donc $\lim\limits_{x\to+\infty}\frac{\sqrt{x+3}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x+1}}=\frac{1}{2}$

► $\lim\limits_{x\to0^+}\frac{\tan x\sqrt{\tan x}-\sin x\sqrt{\sin x}}{x^3\sqrt{x}}$

Pour tout $x\in\left]0;\frac{\pi}{2}\right[$ *; on a :* $\sin x>0$ *et* $\tan x>0$ *; donc* $\sin x=\sqrt{\sin^2 x}$ *et* $\tan x=\sqrt{\tan^2 x}$

D'où $\frac{\tan x\sqrt{\tan x}-\sin x\sqrt{\sin x}}{x^3\sqrt{x}}=\frac{\sqrt{\tan x}^3-\sqrt{\sin x}^3}{x^3\sqrt{x}}$

$$=\frac{\left(\sqrt{\tan x}-\sqrt{\sin x}\right)\left(\tan x+\sin x+\sqrt{\tan x}.\sqrt{\sin x}\right)}{x^3\sqrt{x}}$$

$$= \frac{\left(\sqrt{\tan x} - \sqrt{\sin x}\right)\left(\tan x + \sin x + \sqrt{\sin x \times \tan x}\right)}{x^3\sqrt{x}}$$

$$= \frac{\left(\frac{\sqrt{\sin x}}{\sqrt{\cos x}} - \sqrt{\sin x}\right)\left(\tan x + \sin x + \sqrt{\sin x \times \tan x}\right)}{x^3\sqrt{x}}$$

$$= \frac{\sqrt{\sin x}\left(\frac{1}{\sqrt{\cos x}} - 1\right)\left(\tan x + \sin x + \sqrt{\sin x \times \tan x}\right)}{x^3\sqrt{x}}$$

$$= \frac{\sqrt{\sin x}\left(1 - \sqrt{\cos x}\right)\left(\tan x + \sin x + \sqrt{\sin x \times \tan x}\right)}{x^3\sqrt{x}\left(\sqrt{\cos x}\right)}$$

$$= \frac{\sqrt{\sin x}\left(1 - \sqrt{\cos x}\right)}{x^2\sqrt{x}\left(\sqrt{\cos x}\right)} \times \frac{\left(\tan x + \sin x + \sqrt{\sin x \times \tan x}\right)}{x}$$

$$= \left(\frac{\left(\frac{1-\cos x}{x^2}\right) \times \frac{\sqrt{\sin x}}{\sqrt{x}}}{\left(1+\sqrt{\cos x}\right)\sqrt{\cos x}}\right) \times \left(\frac{\tan x}{x} + \frac{\sin x}{x} + \sqrt{\frac{\sin x}{x} \times \frac{\tan x}{x}}\right)$$

Donc

$$\lim_{x\to 0^+} \frac{\tan x\sqrt{\tan x} - \sin x\sqrt{\sin x}}{x^3\sqrt{x}} = \lim_{x\to 0^+} \left(\frac{\left(\frac{1-\cos x}{x^2}\right) \times \frac{\sqrt{\sin x}}{\sqrt{x}}}{\left(1+\sqrt{\cos x}\right)\sqrt{\cos x}}\right) \times \left(\frac{\tan x}{x} + \frac{\sin x}{x} + \sqrt{\frac{\sin x}{x} \times \frac{\tan x}{x}}\right)$$

$$= \lim_{x\to 0^+} \left(\frac{\left(\frac{1-\cos x}{x^2}\right) \times \sqrt{\frac{\sin x}{x}}}{\left(1+\sqrt{\cos x}\right)\sqrt{\cos x}}\right) \times \left(\frac{\tan x}{x} + \frac{\sin x}{x} + \sqrt{\frac{\sin x}{x} \times \frac{\tan x}{x}}\right)$$

Et $\lim_{x\to 0^+} \left(\frac{1-\cos x}{x^2}\right) = \frac{1}{2}$; $\lim_{x\to 0^+} \frac{\tan x}{x} = \lim_{x\to 0^+} \frac{\sin x}{x} = 1$; $\lim_{x\to 0^+} \left(\frac{1}{\left(1+\sqrt{\cos x}\right)\sqrt{\cos x}}\right) = \frac{1}{2}$

D'où $\lim_{x\to 0^+} \frac{\tan x\sqrt{\tan x} - \sin x\sqrt{\sin x}}{x^3\sqrt{x}} = \frac{3}{4}$

► $\lim_{x\to 0}\dfrac{\sqrt{1-\cos\left(x^2\right)}}{1-\cos x}$

On sait que : $\left(\forall x\in\mathbb{R}\right)$; $1-\cos x>0$

Donc $\dfrac{\sqrt{1-\cos\left(x^2\right)}}{1-\cos x}=\sqrt{\dfrac{1-\cos\left(x^2\right)}{\left(1-\cos x\right)^2}}$

$$=\sqrt{\frac{1-\cos\left(x^2\right)}{x^2}}\times\frac{x^2}{1-\cos x}$$

Et $\lim_{x\to 0}\dfrac{x^2}{1-\cos x}=\lim_{x\to 0}\dfrac{1}{\dfrac{1-\cos x}{x^2}}=2$; $\lim_{x\to 0}\sqrt{\dfrac{1-\cos\left(x^2\right)}{x^2}}=\lim_{t\to 0}\sqrt{\dfrac{1-\cos t}{t^2}}=\dfrac{1}{\sqrt{2}}$

D'où $\lim_{x\to 0}\dfrac{\sqrt{1-\cos\left(x^2\right)}}{1-\cos x}=\lim_{x\to 0}\dfrac{1}{\sqrt{2}}\times 2=\sqrt{2}$

► $\lim_{x\to\frac{\pi}{4}}\left(\tan\left(2x\right).\tan\left(x-\dfrac{\pi}{4}\right)\right)$

On pose $t=x-\dfrac{\pi}{4}$ *; alors* $x\to\dfrac{\pi}{4}\Rightarrow t\to 0$ *et* $2t+\dfrac{\pi}{2}=2x$

On a : $\lim_{x\to\frac{\pi}{4}}\left(\tan\left(2x\right).\tan\left(x-\dfrac{\pi}{4}\right)\right)=\lim_{t\to 0}\left(\tan\left(2t+\dfrac{\pi}{2}\right).\tan\left(t\right)\right)$

$$=\lim_{t\to 0}\left(\frac{1}{\tan\left(2t\right)}.\tan\left(t\right)\right)$$

$$=\lim_{t\to 0}\left(\frac{1}{2}\times\frac{1}{\dfrac{\tan\left(2t\right)}{2t}}.\frac{\tan\left(t\right)}{t}\right)$$

Et $\lim_{t\to 0}\dfrac{\tan\left(2t\right)}{2t}=\lim_{t\to 0}\dfrac{\tan\left(t\right)}{t}=1$

Donc $\lim_{x\to\frac{\pi}{4}}\left(\tan\left(2x\right).\tan\left(x-\dfrac{\pi}{4}\right)\right)=\dfrac{1}{2}$

► $\lim_{|x|\to+\infty}\dfrac{x}{x^2+1}.\cos x$

On sait que pour tout $x\in\mathbb{R}$; $\left|\cos x\right|\le 1$

$$Donc\ \left|\frac{x}{x^2+1}\right|\left|\cos x\right| \le \left|\frac{x}{x^2+1}\right| \Leftrightarrow 0 \le \left|\frac{x}{x^2+1}\cos x\right| \le \left|\frac{x}{x^2+1}\right|$$

$$Et\ \lim_{|x|\to+\infty}\left|\frac{x}{x^2+1}\right| = 0$$

$$D'où\ \lim_{|x|\to+\infty}\left|\frac{x}{x^2+1}\cos x\right| = 0 \Leftrightarrow \lim_{|x|\to+\infty}\frac{x}{x^2+1}\cos x = 0$$

► $\lim_{x\to-1}\dfrac{\sin\left(\cos\left(\frac{\pi}{2}x\right)\right)}{x+1}$.

$$On\ a:\ \frac{\sin\left(\cos\frac{\pi}{2}x\right)}{x+1} = \frac{\sin\left(\cos\left(\frac{\pi}{2}x\right)\right)}{\cos\left(\frac{\pi}{2}x\right)} \times \frac{\cos\left(\frac{\pi}{2}x\right)}{x+1}$$

$$Calculons\ \lim_{x\to-1}\frac{\sin\left(\cos\left(\frac{\pi}{2}x\right)\right)}{\cos\left(\frac{\pi}{2}x\right)}.et\ \lim_{x\to-1}\frac{\cos\left(\frac{\pi}{2}x\right)}{x+1}.$$

● On pose : $X = \cos\left(\frac{\pi}{2}x\right)$; $x \to -1$ alors $X \to 0$

$$D'où\ \lim_{x\to-1}\frac{\sin\left(\cos\left(\frac{\pi}{2}x\right)\right)}{\cos\left(\frac{\pi}{2}x\right)}. = \lim_{X\to0}\frac{\sin(X)}{X} = 1$$

● On pose : $X = x+1 \Rightarrow x = X-1$; $x \to -1$ alors $X \to 0$

$$D'où\ \lim_{x\to-1}\frac{\cos\left(\frac{\pi}{2}x\right)}{x+1} = \lim_{X\to0}\frac{\cos\left(\frac{\pi}{2}(X-1)\right)}{X}$$

$$= \lim_{X\to0}\frac{\cos\left(\frac{\pi}{2}X-\frac{\pi}{2}\right)}{X}$$

$$= \lim_{X\to0}\frac{\sin\left(\frac{\pi}{2}X\right)}{X}$$

$$= \lim_{X \to 0} \left(\frac{\pi}{2} \times \frac{\sin\left(\frac{\pi}{2} X\right)}{\frac{\pi}{2} X} \right)$$

Donc $\lim_{x \to -1} \frac{\cos\left(\frac{\pi}{2} x\right)}{x+1} = \frac{\pi}{2}$

D'où $\lim_{x \to -1} \frac{\sin\left(\cos\left(\frac{\pi}{2} x\right)\right)}{x+1} = \frac{\pi}{2}$

► $\lim_{x \to 0} \frac{\sin\left(\pi\sqrt{\cos x}\right)}{x}$

Pour tout $x \neq 0$ *on a :* $\frac{\sin\left(\pi\sqrt{\cos x}\right)}{x} = \frac{\sin\left(\pi\sqrt{\cos x} - \pi + \pi\right)}{x}$

$$= \frac{\sin\left(\pi\left(\sqrt{\cos x} - 1\right) + \pi\right)}{x}$$

$$= \frac{-\sin\left(\pi\left(\sqrt{\cos x} - 1\right)\right)}{\pi\left(\sqrt{\cos x} - 1\right)} \times \frac{\pi\left(\sqrt{\cos x} - 1\right)}{x}$$

$$= \frac{-\sin\left(\pi\left(\sqrt{\cos x} - 1\right)\right)}{\pi\left(\sqrt{\cos x} - 1\right)} \times \frac{(\cos x - 1)}{x^2} \times \frac{x}{\sqrt{\cos x} + 1}$$

Et ● $\lim_{x \to 0} \frac{-\sin\left(\pi\left(\sqrt{\cos x} - 1\right)\right)}{\pi\left(\sqrt{\cos x} - 1\right)} = \lim_{X \to 0} \frac{-\sin(X)}{X} = -1$

En posant $X = \pi\left(\sqrt{\cos x} - 1\right)$ *;* $x \to 0$ *alors* $X \to 0$

● $\lim_{x \to 0} \frac{(\cos x - 1)}{x^2} = \frac{1}{2}$

● $\lim_{x \to 0} \frac{x}{\sqrt{\cos x} + 1} = 0$

Donc $\lim_{x \to 0} \frac{\sin\left(\pi\sqrt{\cos x}\right)}{x} = 0$

► $\lim_{x\to\frac{\pi}{3}} \dfrac{\sin x-\sqrt{3}\cos x}{x-\frac{\pi}{3}}$

Pour tout $x\neq\frac{\pi}{3}$ *on a :* $\dfrac{\sin x-\sqrt{3}\cos x}{x-\frac{\pi}{3}}=\dfrac{2\left(\frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\cos x\right)}{x-\frac{\pi}{3}}$

$$=\dfrac{2\left(\cos\left(\frac{\pi}{3}\right)\sin x-\sin\left(\frac{\pi}{3}\right)\cos x\right)}{x-\frac{\pi}{3}}$$

$$=\dfrac{2\sin\left(x-\frac{\pi}{3}\right)}{x-\frac{\pi}{3}}$$

On pose : $X=x-\frac{\pi}{3}$; $x\to\frac{\pi}{3}$ *alors* $X\to 0$

Donc $\lim_{x\to\frac{\pi}{3}} \dfrac{\sin x-\sqrt{3}\cos x}{x-\frac{\pi}{3}}=\lim_{x\to\frac{\pi}{3}} \dfrac{2\sin\left(x-\frac{\pi}{3}\right)}{x-\frac{\pi}{3}}$

$$=\lim_{X\to 0}\dfrac{2\sin(X)}{X}=2$$

Par suite $\lim_{x\to\frac{\pi}{3}} \dfrac{\sin x-\sqrt{3}\cos x}{x-\frac{\pi}{3}}=2$

► $\lim_{x\to a}\dfrac{ax^n-a^nx}{x-a}$ *où* $a\in\mathbb{R}$

On considère la fonction h définie sur $\mathbb{R}$ *par :* $h(x)=ax^n-a^nx$

H est une fonction polynôme donc dérivable sur $\mathbb{R}$ *; et* $(\forall x\in\mathbb{R})$; $h'(x)=anx^{n-1}-a^n$

Et $h(a)=0$

On a : $\lim_{x\to a}\dfrac{ax^n-a^nx}{x-a}=\lim_{x\to a}\dfrac{h(x)-h(a)}{x-a}$

$$=h'(a)$$

$$= ana^{n-1} - a^n$$
$$= (n-1)a^n$$

Donc $\lim_{x\to a} \frac{ax^n - a^n x}{x-a} = (n-1)a^n$

► $\lim_{x\to 2} \frac{\sum_{k=1}^{n} x^k - 2^{n+1} + 2}{(3-x)^{n+1} - 1}$

Pour tout $n \in \mathbb{N}$ *; on a :* $\sum_{k=1}^{n} x^k = x \times \frac{x^n - 1}{x-1}$

$$= \frac{x^{n+1} - x}{x-1}$$

(somme des n premiers termes d'une suite géométrique de raison x)

Donc $\sum_{k=1}^{n} x^k - 2^{n+1} + 2 = \frac{x^{n+1} - x}{x-1} - 2^{n+1} + 2$

$$= \frac{x^{n+1} - x}{x-1} - \left(2^{n+1} - 2\right)$$
$$= \frac{x^{n+1} - x - \left(2^{n+1} - 2\right)(x-1)}{x-1}$$
$$= \frac{x^{n+1} - x - \left(2^{n+1}x - 2^{n+1} - 2x + 1\right)}{x-1}$$
$$= \frac{x^{n+1} - 2^{n+1} - \left(2^{n+1}x - 2 \times 2^{n+1} + 2 - x\right)}{x-1}$$
$$= \frac{\left(x^{n+1} - 2^{n+1}\right) - \left(2^{n+1} - 1\right)(x-2)}{x-1}$$
$$= \frac{(x-2)\left(x^n + 2x^{n-1} + \cdots + 2^n\right) - \left(2^{n+1} - 1\right)(x-2)}{x-1}$$
$$= \frac{(x-2)\left(x^n + 2x^{n-1} + \cdots + 2^n\right) - \left(2^{n+1} - 1\right)(x-2)}{x-1}$$
$$= (x-2)\left(\frac{\left(x^n + 2x^{n-1} + \cdots + 2^n\right) - \left(2^{n+1} - 1\right)}{x-1}\right)$$

Et on a : $(3-x)^{n+1} - 1 = \left((3-x) - 1\right)\left((3-x)^n + (3-x)^{n-1} + \cdots + 1\right)$

$$= -(x-2)\left((3-x)^n + (3-x)^{n-1} + \cdots + 1\right)$$

$$Donc\ \lim_{x\to 2}\frac{\sum_{k=1}^{n}x^k-2^{n+1}+2}{(3-x)^{n+1}-1}=\lim_{x\to 2}\frac{(x-2)\left(\frac{\left(x^n+2x^{n-1}+\cdots+2^n\right)-\left(2^{n+1}-1\right)}{x-1}\right)}{-(x-2)\left((3-x)^n+(3-x)^{n-1}+\cdots+1\right)}$$

$$=\lim_{x\to 2}\frac{\left(x^n+2x^{n-1}+\cdots+2^n\right)-\left(2^{n+1}-1\right)}{(1-x)\left((3-x)^n+(3-x)^{n-1}+\cdots+1\right)}$$

$$=\frac{\left(2^n+2^n+\cdots+2^n\right)-\left(2^{n+1}-1\right)}{-\left(1^n+1^{n-1}+\cdots+1\right)}$$

$$=-\frac{2^n(n+1)-\left(2^{n+1}-1\right)}{(n+1)}$$

$$=-\frac{2^n(n+1)-2\times 2^n+1}{(n+1)}$$

$$=-\frac{(n-1)2^n+1}{(n+1)}$$

► $\lim_{x\to 1}\frac{x^{4n+1}\sqrt{x+3}+2\sqrt{x}-4}{1-x^{3n+2}}$

Soit $x\in\mathbb{R}_+^*$ *; on a :* $x^{4n+1}\sqrt{x+3}+2\sqrt{x}-4=x^{4n+1}\left(\sqrt{x+3}-2\right)+2x^{4n+1}-2+2\sqrt{x}-2$

$$=x^{4n+1}\left(\sqrt{x+3}-2\right)+2\left(x^{4n+1}-1\right)+2\left(\sqrt{x}-1\right)$$

$$=x^{4n+1}\left(\frac{x-1}{\sqrt{x+3}+2}\right)+2(x-1)\sum_{k=0}^{4n}x^k+2\left(\frac{x-1}{\sqrt{x}+1}\right)$$

$$=(x-1)\left(\frac{x^{4n+1}}{\sqrt{x+3}+2}+2\sum_{k=0}^{4n}x^k+\frac{2}{\sqrt{x}+1}\right)$$

Et $1-x^{3n+2}=-(x-1)\sum_{k=0}^{3n+1}x^k$

$$D'où :\ \lim_{x\to 1}\frac{x^{4n+1}\sqrt{x+3}+2\sqrt{x}-4}{1-x^{3n+2}}=\lim_{x\to 1}\frac{(x-1)\left(\frac{x^{4n+1}}{\sqrt{x+3}+2}+2\sum_{k=0}^{4n}x^k+\frac{2}{\sqrt{x}+1}\right)}{-(x-1)\sum_{k=0}^{3n+1}x^k}$$

$$=\lim_{x\to 1} -\left(\frac{\dfrac{x^{4n+1}}{\sqrt{x+3}+2}+2\sum_{k=0}^{4n}x^k+\dfrac{2}{\sqrt{x}+1}}{\sum_{k=0}^{3n+1}x^k}\right)$$

Or $\lim_{x\to 1}\frac{x^{4n+1}}{\sqrt{x+3}+2}=\frac{1}{4}$; $\lim_{x\to 1}2\sum_{k=0}^{4n}x^k=2(4n+1)$; $\lim_{x\to 1}\frac{2}{\sqrt{x}+1}=1$ *et* $\lim_{x\to 1}\sum_{k=0}^{3n+1}x^k=3n+2$

Donc $\lim_{x\to 1}\frac{x^{4n+1}\sqrt{x+3}+2\sqrt{x}-4}{1-x^{3n+2}}=-\frac{\frac{1}{4}+8n+2+1}{3n+2}$

$$=-\frac{1+32n+8+4}{4(3n+2)}$$

$$=-\frac{32n+13}{4(3n+2)}$$